9.5: The Logic of Relations

So far we've only considered WFFs containing *monadic* predicates like Ax and $(\exists y)(Py \rightarrow Qy)$, where monadic predicates stand for verb phrases like 'is happy' or 'is a woman'. However, predicate logic also contains binary predicates (i.e., 2-place predicates) for forming WFFs like Fyz and $\sim(x)(Ax \rightarrow (\exists y)(By \cdot Gxy))$; ternary predicates for forming WFFs like $\sim\mathrm{Rabc}$; and in general, n-place predicates for all n. Binary predicates stand for expressions like 'father of' and 'shorter than' that express *relations* between individuals. In this section we will consider arguments and symbolizations involving n-place predicates. Proofs work no differently with n-place predicates than with monadic predicates. The major trick is translation.

<u>Example</u> (from text)

Al is taller than Bob. Bob is taller than Chris. If one thing is taller than a second, and the second is taller than a third, then the first is taller than the third. So, Al is taller than Chris. (Txy : x is taller than y; a: Al; b: Bob; c: Chris)

Translation: $\mathrm{Tab}.\ \mathrm{Tbc}.\ (x)(y)(z)[(Txy \cdot Tyz) \rightarrow Txz] \therefore \mathrm{Tac}$

1.	Tab	
2.	Tbc	
3.	$(x)(y)(z)[(Txy \cdot Tyz) \rightarrow Txz]$	$\therefore \mathrm{Tac}$
4.	$(y)(z)[(Tay \cdot Tyz) \rightarrow Taz]$	3 UI
5.	$(z)[(Tab \cdot Tbz) \rightarrow Taz]$	4 UI
6.	$(Tab \cdot Tbc) \rightarrow Tac$	5 UI
7.	$(Tab \cdot Tbc)$	1,2 Conj
8.	Tac	6,7 MP

Translation with 2-place predicates

Consider the following scheme of abbreviation:

Gx:	x is a genius
Axy:	x admires y
Wxy:	x works for y
b:	Kim
p:	Pat
c:	Michael

Now let us translate the following sentences according to this scheme.

Kim admires Pat.

Pat admires Michael, but Kim doesn't.

Pat and Michael admire each other.

Even though Michael isn't a genius, Pat still admires him.

Anyone who works for Pat is a genius.

Kim admires no one.

or

Pat hires anyone who admires her.

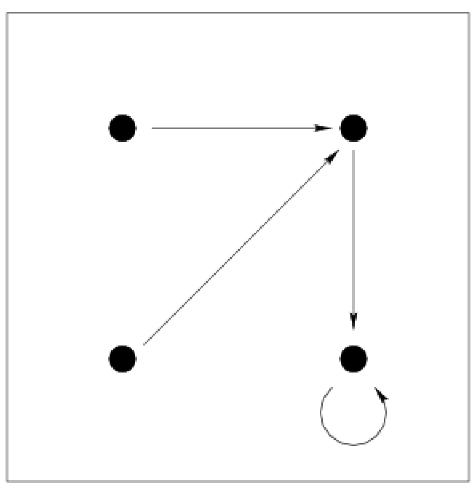
Someone Pat admires is a genius.

If Pat admires someone, she hires them.

Kim is a genius. So, anyone who hires Kim hires a genius.

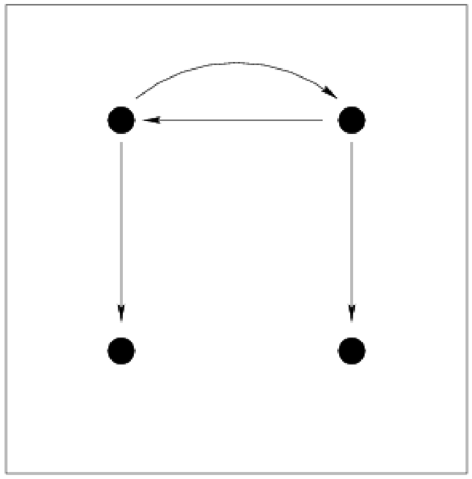
More on 2-place Relations

The following material is not in the text. Its purpose is to illustrate the importance of the order of the universal and existential quantifiers when they occur together, and also to illustrate the importance of the order of the arguments in an atomic sentence. We will do so by building a series of small "models" in which certain sentences are true. Many sentences are true in each model, of course. We will just identify the ones that are particularly salient. In these models, dots represent persons, and an arrow between two dots, i.e., $\bullet \longrightarrow \bullet$, indicates that the person represented by the first dot loves the person represented by the second. The answers are left out of the notes so that we can construct them as exercises in class.



The Caring World

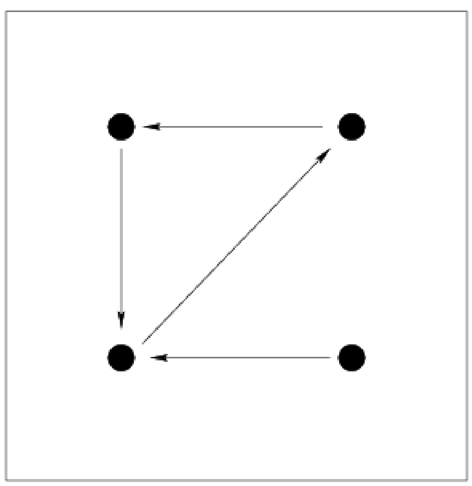
English:
Logic:



The Cared-for World

English:

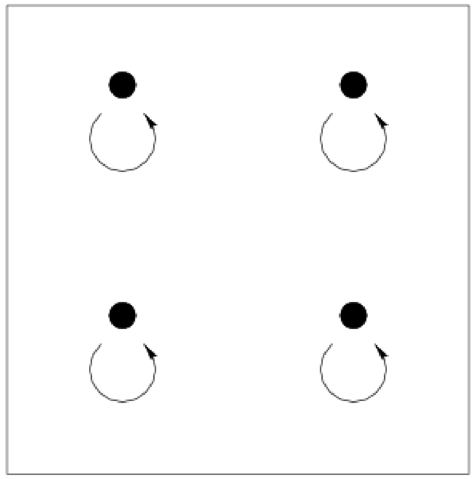
Logic:



Unrequited Love

English:

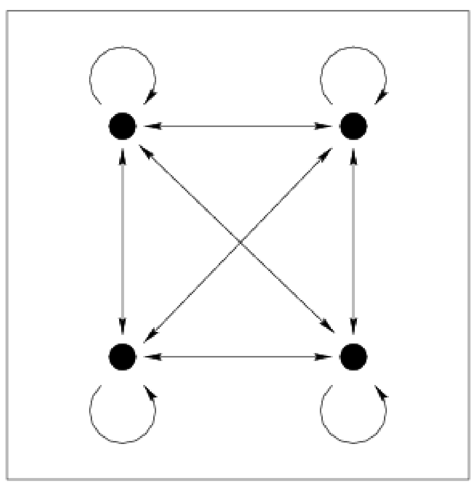
Logic:



The Narcissistic World

English:

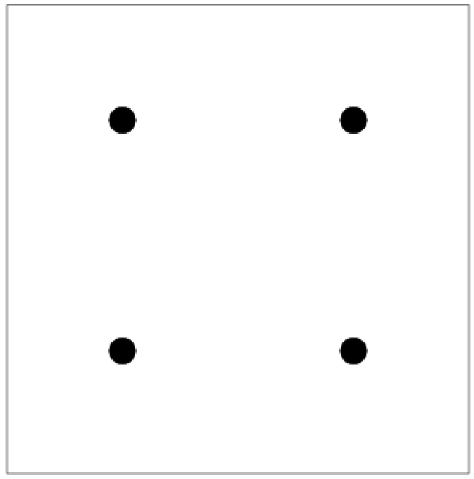
Logic:



The 60s

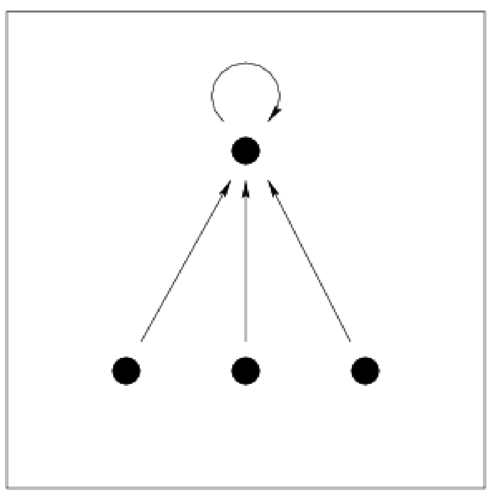
English:

Logic:



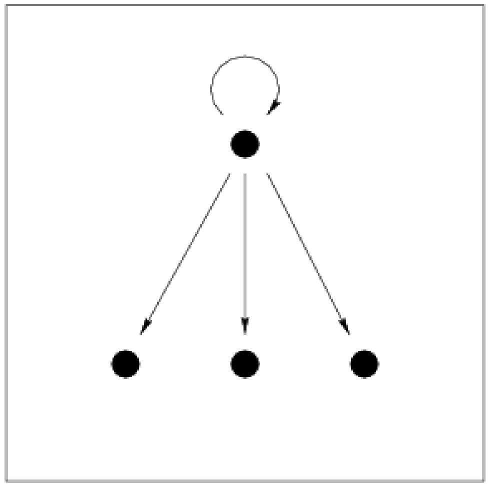
The Bleak World

English:
Logic:



The Universal Object of Adoration

English:
Logic:



The Universal Adorer

English:
Logic:

Important Properties of Relations

R is *symmetrical* = if x bears R to y, then y bears R to x (i.e., (x)(y)(Rxy → Ryx).

Example

being a sibling of

R is *asymmetrical* = if x bears R to y, then y does not bear R to x (i.e., (x)(y)(Rxy → ~Ryx).

Example

mother of

R is *nonsymmetrical* = R is neither symmetrical nor asymmetrical

Example

loves

R is *reflexive* = x bears R to x, for all x (i.e., $(x)Rxx$).

Example

identical to

R is *irreflexive* = x does not bear R to x, for any x (i.e., $(x)\sim Rxx$).

Example

grandmother of, sister of

R is *transitive* = if x bears R to y, and y bears R to z, then x bears R to z (i.e., $(x)(y)(z)[(Rxy \bullet Ryz) \rightarrow Rxz]$).

Example

*ancestor of, taller than, (*not *sister of!)*

R is *intransitive* = if x bears R to y, and y bears R to z, then x does not bear R to z (i.e., $(x)(y)(z)[(Rxy \bullet Ryz) \rightarrow \sim Rxz]$).

Example

father of

R is *nontransitive* = R is neither transitive nor intransitive

Example

sister of?