8.5: Reductio Ad Absurdum

Suppose you were given as a premise that statement A implies the contradiction $B \bullet \sim B$, i.e., that $A \rightarrow (B \bullet \sim B)$. Using CP and MT it is easy to show that, from this premise, $\sim A$ follows:

1.	$A \rightarrow (B \bullet \sim B)$	
2.	B	Assume
3.	$\sim\sim B$	1,2 Conj
4.	$B \rightarrow \sim\sim B$	2-3 CP
5.	$\sim B \vee \sim\sim B$	4 MI
6.	$\sim(B \bullet \sim B)$	5 DeM
7.	$\sim A$	1-6 MT

The basic insight illustrated here is this:

Whatever implies a contradiction must be false!

This principle is embodied in a proof method known as *reductio ad absurdum* (RAA), which we now add to our system of propositional logic.

The Two General Forms of RAA

(*Premises and derived statements*)

⋮	⋮	
$m.$	p	Assume
⋮	⋮	
$n.$	$q \bullet \sim q$	…
n+1.	$\sim p$	m-n RAA

(*Premises and derived statements*)

⋮	⋮	
$m.$	$\sim p$	Assume
⋮	⋮	
$n.$	$q \bullet \sim q$	…
n+1.	p	m-n RAA

NB: When the conclusion of an argument is the *negation* of a statement, your assumption should be the *unnegated* portion of the statement.

Example 1

1. $B \leftrightarrow \sim A$
2. $\sim A \rightarrow \sim C$
3. $C \vee D$
4. $\sim C \rightarrow \sim D$ $\quad\therefore\ \sim B$

Note that it wouldn't have been a *mistake* to assume $\sim\sim B$ in line 5, but it would have added an unnecessary step, since you would first need to apply DN to derive B before you could infer $\sim A$ as in line 8.

Tip 12: If the conclusion of an argument (or, more generally, any statement you are trying to prove in the course of an argument) is not a conditional statement, and a direct proof of the statement looks long or difficult, try RAA.

Example 2

1. $\sim A \rightarrow [(B \bullet D) \vee (B \bullet G)]$
2. $(A \rightarrow E) \bullet (\sim B \vee F)$
3. $\sim K \rightarrow (\sim E \bullet \sim F) \qquad \therefore\ K \vee D$

Here is a solution to the preceding problem.

1.	$\sim A \rightarrow [(B \bullet D) \vee (B \bullet G)]$	
2.	$(A \rightarrow E) \bullet (\sim B \vee F)$	
3.	$\sim K \rightarrow (\sim E \bullet \sim F)$	$\therefore$ $K \vee D$
4.	$\sim(K \vee D)$	Assume [RAA]
5.	$\sim K \bullet \sim D$	4 DeM
6.	$\sim K$	5 Simp
7.	$\sim E \bullet \sim F$	3, 6 MP
8.	$\sim E$	7 Simp
9.	$A \rightarrow E$	2 Simp
10.	$\sim A$	8, 9 MT
11.	$(B \bullet D) \vee (B \bullet G)$	1, 10 MP
12.	$B \bullet (D \vee G)$	11 Dist
13.	B	12 Simp
14.	$\sim B \vee F$	2 Simp
15.	$\sim F$	7 Simp
16.	$\sim B$	14, 15 DS
17.	$B \bullet \sim B$	13, 16 Conj
18.	$K \vee D$	4-17 RAA

We could in principle dispense with CP and just use RAA. To illustrate:

Example 3

1. $Z \rightarrow (\sim Y \rightarrow X)$
2. $Z \rightarrow \sim Y \qquad \therefore\ Z \rightarrow X$

It is often useful (even necessary) to nest an RAA subproof within a CP. (I will definitely put a problem like this one on the next exam!)

Example 4

1. $A \rightarrow (B \vee C)$
2. $D \rightarrow \sim C \qquad \therefore A \rightarrow \sim(D \bullet \sim B)$