8.5: Reductio Ad Absurdum

Suppose you were given as a premise that statement A implies the contradiction $B \bullet \sim B$, i.e., that $A \to (B \bullet \sim B)$. Using CP and MT it is easy to show that, from this premise, $\sim A$ follows:

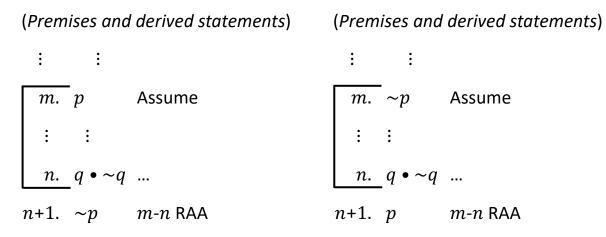
1. $A \rightarrow (B \bullet \sim B)$	
2. B	Assume
3 . ~~B	1,2 Conj
4. $B \rightarrow \sim \sim B$	2-3 CP
5. $\sim B \lor \sim \sim B$	4 MI
6. ~(B • ~B)	5 DeM
7. ~A	1-6 MT

The basic insight illustrated here is this:

Whatever implies a contradiction must be false!

This principle is embodied in a proof method known as *reductio ad absurdum* (RAA), which we now add to our system of propositional logic.

The Two General Forms of RAA



NB: When the conclusion of an argument is the *negation* of a statement, your assumption should be the *unnegated* portion of the statement.

Example 1

1.
$$B \leftrightarrow \sim A$$

2. $\sim A \rightarrow \sim C$
3. $C \lor D$
4. $\sim C \rightarrow \sim D$ $\therefore \sim B$

Note that it wouldn't have been a *mistake* to assume $\sim \sim B$ in line 5, but it would have added an unnecessary step, since you would first need to apply DN to derive B before you could infer $\sim A$ as in line 8.

Tip 12: If the conclusion of an argument (or, more generally, any statement you are trying to prove in the course of an argument) is not a conditional statement, and a direct proof of the statement looks long or difficult, try RAA.

Example 2

1.
$$\sim A \rightarrow [(B \bullet D) \lor (B \bullet G)]$$

2. $(A \rightarrow E) \bullet (\sim B \lor F)$
3. $\sim K \rightarrow (\sim E \bullet \sim F) \quad \therefore K \lor D$

Here is a solution to the preceding problem.

1.
$$\sim A \rightarrow [(B \bullet D) \lor (B \bullet G)]$$
2. $(A \rightarrow E) \bullet (\sim B \lor F)$ 3. $\sim K \rightarrow (\sim E \bullet \sim F)$ $\therefore K \lor D$ 4. $\sim (K \lor D)$ Assume [RAA]5. $\sim K \bullet \sim D$ 4 DeM6. $\sim K$ 5 Simp7. $\sim E \bullet \sim F$ 3, 6 MP8. $\sim E$ 7 Simp9. $A \rightarrow E$ 2 Simp10. $\sim A$ 8, 9 MT11. $(B \bullet D) \lor (B \bullet G)$ 1, 10 MP12. $B \bullet (D \lor G)$ 11 Dist13. B 12 Simp14. $\sim B \lor F$ 2 Simp15. $\sim F$ 7 Simp16. $\sim B$ 14, 15 DS17. $B \bullet \sim B$ 13, 16 Conj18. $K \lor D$ 4-17 RAA

We could in principle dispense with CP and just use RAA. To illustrate:

Example 3

1.
$$Z \rightarrow (\sim Y \rightarrow X)$$

2. $Z \rightarrow \sim Y$ $\therefore Z \rightarrow X$

It is often useful (even necessary) to nest an RAA subproof within a CP. (I will definitely put a problem like this one on the next exam!)

Example 4

1.
$$A \rightarrow (B \lor C)$$

2. $D \rightarrow \sim C$ $\therefore A \rightarrow \sim (D \bullet \sim B)$