

8.4: Conditional Proof

By the rule of conjunction, if we are given A and B , we can infer $(A \bullet B)$. Suppose, however, that all we are given is A , and that we don't know anything about B . In this case, we recognize (just as we recognize the validity of the rule of conjunction), that it is valid to infer that *if* B is true, then $(A \bullet B)$ is true. That is, we recognize that the following argument is valid:

$$A \therefore B \rightarrow (A \bullet B)$$

However, despite their number, it is not possible to demonstrate that this argument is valid using our 18 rules. Rather, we need an additional technique known as *conditional proof* ("CP").

The idea: you can prove a conditional $p \rightarrow q$ by *assuming* the antecedent p is true (hence the "conditional" in "conditional proof") and then deriving the consequent q from this assumption (using, as necessary, the 18 rules). The use of CP is indicated in the following proof of the valid argument above:

1.	A	
2.	B	Assume
3.	$A \bullet B$	1,2 Conj
4.	$B \rightarrow (A \bullet B)$	2-3 CP

The part of the proof that relies upon the assumption (known as a *subproof*, and also as the *scope* of the assumption on the first line of the subproof, i.e., in this case, B) is "boxed in" as indicated. Notice that the justification in line 4 refers to the entire subproof: we write "2-3" to indicate "everything between lines 2 and 3, inclusive". In this case, of course, that only means lines 2 and 3 themselves, but examples below will indicate longer subproofs in which any number of lines separate the assumption from the final line in the subproof.

The General Form of CP

(Premises and derived statements)

	∴	∴	
┌	$m.$	p	Assume
	∴	∴	
	$n.$	q	...
└	$n+1.$	$p \rightarrow q$	$m-n$ CP

Two things to note:

- $p \rightarrow q$ here needn't be the conclusion of the argument; it might only be a WFF that you need on your way to the conclusion.
- One or more instances of CP might be nested within another; that is, in terms of the general form, there might be other occurrences of CP between lines m and n .

Tip 11 (IMPORTANT!): If the conclusion of an argument (or, more generally, any statement you are trying to prove in the course of an argument) is a conditional statement, use CP.

A Bunch o' Examples

Example 1

1. $\sim S \rightarrow W$
2. $\sim R \rightarrow U$
3. $(U \vee W) \rightarrow T \quad \therefore \sim(S \bullet R) \rightarrow (T \vee Z)$

The following example illustrates how one subproof can be nested within another.

Example 2

$$1. S \rightarrow (T \rightarrow R)$$

$$2. (T \bullet R) \rightarrow L \quad \therefore S \rightarrow (T \rightarrow L)$$

Example 3

1. $S \rightarrow (G \vee F)$
2. $F \rightarrow W$
3. $\sim W \quad \therefore \sim S \vee G$

NB: A box in a proof indicates the scope the initial assumption, that is, how long the assumption is in effect. Once a box has been “closed”, *nothing within the box can be used in the rest of the proof* (unless it also appears outside the box), since everything in the box was derived under an assumption that is no longer being made.

The following example illustrates how one might use CP in an argument whose conclusion is not a conditional.

Example 4

1. $(B \vee A) \rightarrow C$
2. $A \rightarrow \sim C$
3. $\sim A \rightarrow B \quad \therefore B \leftrightarrow C$