

8.3 Five More Equivalence Rules

In this section the remaining five equivalence rules are introduced.

Rule 14: Distribution (Dist)

$$\begin{aligned} p \bullet (q \vee r) &:: (p \bullet q) \vee (p \bullet r) \\ p \vee (q \bullet r) &:: (p \vee q) \bullet (p \vee r) \end{aligned}$$

Example: A proof for $F \vee (G \bullet L), (F \vee L) \rightarrow D \therefore D$

1. $F \vee (G \bullet L)$
2. $(F \vee L) \rightarrow D \therefore D$

Rule 15: Exportation (Exp)

$$(p \bullet q) \rightarrow r :: p \rightarrow (q \rightarrow r)$$

Example: A proof for $(\sim W \bullet D) \rightarrow \sim J, \sim W \therefore D \rightarrow \sim J$

1. $(\sim W \bullet D) \rightarrow \sim J$
2. $\sim W \therefore D \rightarrow \sim J$

Rule 16: Redundancy (Re)

$$p :: p \bullet p$$

$$p :: p \vee p$$

Example: A proof for $R \vee I, R \rightarrow B, I \rightarrow B \therefore B$

1. $R \vee I$
2. $R \rightarrow B$
3. $I \rightarrow B \therefore B$

Rule 17: Material Equivalence (ME)

$$p \leftrightarrow q :: (p \rightarrow q) \bullet (q \rightarrow p)$$

$$p \leftrightarrow q :: (p \bullet q) \vee (\sim p \bullet \sim q)$$

Example: A proof for $W \leftrightarrow (L \vee F), B, B \rightarrow \sim L, \sim F \therefore \sim W$

1. $W \leftrightarrow (L \vee F)$
2. B
3. $B \rightarrow \sim L$
4. $\sim F \therefore \sim W$

Rule 18: Material Implication (MI)

$$p \rightarrow q :: \sim p \vee q$$

Example: A proof for $(\sim N \vee E) \rightarrow \sim S \therefore \sim N \rightarrow \sim S$

1. $(\sim N \vee E) \rightarrow \sim S \therefore \sim N \rightarrow \sim S$

More Tips

Tip 8: Material implication can lead to useful applications of distribution.

See above proof.

Tip 9: Distribution can lead to useful applications of simplification.

See above proof.

Tip 10: Addition can lead to useful applications of material implication.

Examples:

$$1. B \quad \therefore A \rightarrow B$$

$$1. \sim F \quad \therefore F \rightarrow G$$