7.5 Tautology, Contradiction, Contingency, and Logical Equivalence

Definition : A compound statement is a *tautology* if it is true regardless of the truth values assigned to its component atomic statements.

Equivalently, in terms of truth tables:

Definition: A compound statement is a *tautology* if there is a **T** beneath its main connective in every row of its truth table.

Examples

Either it is raining or it is not raining.

R	R	$\vee$	$\sim$ R
T	T	T	F
F	F	T	T

If Socrates is a philosopher and a windbag, then Socrates is a philosopher.

S	W	(S $\bullet$ W)	$\rightarrow$	S
T	T	T	T	T
T	F	F	T	T
F	T	F	T	F
F	F	F	T	F

If Willard is either a philosopher or a windbag but he's not a philosopher, and then Willard is a windbag.

P	W	$((P \vee W)$	$\bullet$	$\sim P)$	$\rightarrow$	W
T	T	T	F	F	T	T
T	F	T	F	F	T	F
F	T	T	T	T	T	T
F	F	F	F	T	T	F

Definition: A compound statement is a *contradiction* if it is false regardless of the truth values assigned to its component atomic statements.

Equivalently, in terms of truth tables:

Definition: A compound statement is a *contradiction* if there is an **F** beneath its main connective in every row of its truth table.

Examples

It is raining and it is not raining.

R	R	$\bullet$	$\sim R$
T	T	F	F
F	F	F	T

Willard is either a philosopher or a windbag, and he's neither a philosopher nor a windbag.

P	W	(P ∨ W)	•	(∼P	•	∼W)
T	T	T	F	F	F	F
T	F	T	F	F	F	T
F	T	T	F	T	F	F
F	F	F	F	T	T	T

Definition: A compound statement is a *contingent* if it is true on some assignments of truth values to its component atomic statements, and false on others.

Equivalently, in terms of truth tables:

Definition: A compound statement is a *contingent* if there is **T** beneath its main connective in at least one row of its truth table, and an **F** beneath its main connective in at least one row of its truth table.

Example

Willard is either a philosopher or a windbag, and he's not a philosopher .

P	W	(P ∨ W)	•	∼P
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	F	T

Definition: Two statements are *logically equivalent* if they have the same truth values regardless of the truth values assigned to their atomic components.

Equivalently, in terms of truth tables:

Definition: Two statements are *logically equivalent* if, in a truth table for both statements, the same truth value occurs beneath the main connectives of the two statements in each row.

Example

"It is false that Willard is either a philosopher or a linguist" and "Willard is not a philosopher and he is not a linguist"

P	L	$\sim$	$(P \vee L)$	$(\sim P$	$\bullet$	$\sim L)$
T	T	F	T	F	F	F
T	F	F	T	F	F	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T

Recall from the truth table schema for $\leftrightarrow$ that a biconditional $\alpha \leftrightarrow \beta$ is true just in case α and β have the same truth value. This fact yields a further alternative definition of logical equivalence in terms of truth tables:

Definition: Two statements α and β are *logically equivalent* if the biconditional statement $\alpha \leftrightarrow \beta$ is a tautology.

Example

Simply consider the truth table for the example above when we form a biconditional out of the two statements:

P	L	$\sim$	$(P \vee L)$	$\leftrightarrow$	$(\sim P$	$\bullet$	$\sim L)$
T	T	F	T	T	F	F	F
T	F	F	T	T	F	F	T
F	T	F	T	T	T	F	F
F	F	T	F	T	T	T	T