7.3 Using Truth Tables to Evaluate Arguments

Consider first the following valid argument.

1. Either Israel will abandon the West Bank settlements or the suicide bombings will continue. But Israel will never abandon the West Bank settlements. Therefore, the suicide bombings will continue. (A: Israel will abandon the West Bank settlements; B: The suicide bombings will continue.)

We can symbolize this argument as follows:

2. $(A \vee B),\ \sim A\ \therefore\ B$

We want to use truth tables to verify that this argument is valid. To do so, we first construct the truth table. Then we use it to test for validity.

Constructing the Truth Table

1. Begin by listing the statement letters that occur in the (symbolization of) the argument in the order in which they first appear followed by the argument thus:

A	B	$A \vee B$,	$\sim A$	$\therefore$	B

2. Now add all the possible truth value assignments there are for A and B. In any truth table for a sentence or argument, there will always be 2^n, where n is the number of statement letters in the sentence or argument. So there will be 4 rows in this truth table. To construct the truth value assignments begin in the column to the left of the vertical bar and alternate Ts and Fs:

A	B	A ∨ B, ∼A ∴ B
	T	
	F	
	T	
	F	

3. Then move to the next column to the left and alternate pairs of Ts and Fs:

A	B	A ∨ B, ∼A ∴ B
T	T	
T	F	
F	T	
F	F	

Continue in this fashion until you have truth values beneath every sentence letter (as we do already in this example).

4. Fill in the truth table:

A	B	$A \vee B$,	$\sim$A	$\therefore$ B
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Discussion: How do we know under which elements of an argument to put truth values? In actual practice, it is up to you, so long as there is enough information for you to determine the validity of the argument. But it tends to look best if you fill in truth values according to the following two principles (these principles *must* be followed when you are using the Web Tutor):\

- Put truth values beneath every occurrence of a logical operator.

- Put truth values beneath an occurrence of a statement letter *if and only if* that occurrence is either a premise or the conclusion of the argument.

Evaluating the argument

Definition: Let p be a WFF occurring as a premise or the conclusion in an argument, and let **R** be a row of a truth table for that argument. Then p is said to be *true in **R*** iff either (i) p is atomic and 'T' occurs beneath p in **R**, or (ii) p is compound and 'T' occurs beneath its main logical operator in **R**. Otherwise, p is *false in **R**.*

There are two simple steps for determining whether or not an argument is valid:

1. Look for a row of the truth table in which the premises are *ALL* true and the conclusion is false.

2. If there is no such a row, the argument is *valid*; if there is, it is *invalid*.

Looking at the truth table above, we see that it is indeed valid, as expected.

Discussion: By definition, a valid argument is one that cannot have true premises and a false conclusion. That is, more picturesquely, there is no "possible world" in which the premises are true and the conclusion false. Now, each row of a truth table represents a possible world, or rather, a class of possible worlds, namely, all of those worlds in which the statement letters have the truth values assigned them in that row. Since the statements in an argument are all truth functional, each row shows what the truth values of the premises and the conclusion would have been if the statement letters had had the truth values assigned to them in that row. Since we have listed *all* possible truth value assignments, the truth table, in effect, shows what all the truth values that the premises and conclusion could possibly have had. Hence, if the argument is valid — i.e., if there is no possible in which the premises are true and the conclusion false — there should be no row of the truth table in which the premises are assigned T and the conclusion F.

Another Example

Consider the following argument:

1. Abortion is permissible only if fetuses are not innocent human beings or it is not always wrong to kill innocent human beings. But it is always wrong to killl innocent human beings. So abortion is not permissible. (A: Abortion is permissible; B: Fetuses are innocent human beings; W: It is always wrong to kill innocent human beings.)

We can symbolize this argument as follows:

2. $A \rightarrow (\sim B \vee \sim W),\ W\ \therefore\ \sim A$

A	B	W	A	$\rightarrow$	$(\sim$B	$\vee$	$\sim$W),	W	$\therefore$	$\sim$A
T	T	T								
T	T	F								
T	F	T								
T	F	F								
F	T	T								
F	T	F								
F	F	T								
F	F	F								

The completed truth table looks like this:

A	B	W	A	$\rightarrow$	$(\sim$B	$\vee$	$\sim$W),	W	$\therefore$	$\sim$A
T	T	T		F	F	F	F	T		F
T	T	F		T	F	T	T	F		F
T	F	T		T	T	T	F	T		F
T	F	F		T	T	T	T	F		F
F	T	T		T	F	F	F	T		T
F	T	F		T	F	T	T	F		T
F	F	T		T	T	T	F	T		T
F	F	F		T	T	T	T	F		T

And we can see from row 3 that it is invalid: the premises are true but the conclusion is false when A is true, B is false, and W is true.