Chapter 7: Truth Tables

Introduction

- Chapter 6 provided us with useful methods for evaluating categorical syllogisms.

- But not all arguments are composed of categorical statements:

  1. If God exists, then there there must a good explanation for the existence of evil.
  2. God exists.
  3. So, there must be a good explanation for the existence of evil.

- The method of Venn Diagrams does not help us to assess the validity of this obviously valid argument.

  - It's an instance of a famous argument form called *modus ponens*, with which you will become very familiar!

- Chapters 7-9 will introduce us to general methods for evaluating arguments like the one above as well as categorical syllogisms.

- The current chapter introduces us to *the method of truth tables*.

- To apply the method, we must first learn how to translate English sentences into *symbols*.

- This will enable us to apply the methods introduced in this chapter and the chapters to follow.

7.1 — Symbolizing English Arguments

> An *atomic statement* is a statement that does not have any other statement as a component.

1. Shakespeare wrote *Hamlet*.
2. China has a large population.
3. Roses are red.

> An *compound statement* is a statement that has at least one atomic statement as a component.

4. It is not the case that Ben Jonson wrote *Hamlet*.
5. China has a large population and Luxembourg has a small population.
6. Either Palermo is the capital of Sicily or Messina is the capital of Sicily.
7. If Sheboygan is in Wisconsin, then Sheboygan is in the USA.
8. The Democrats win if and only if the Republicans quarrel.

We can symbolize the *atomic* statements in these compounds with capital letters in a *scheme of abbreviation* like the following:

B: Ben Jonson wrote *Hamlet*.
C: China has a large population.
L: Luxembourg has a small population.
P: Palermo is the capital of Sicily.
M: Messina is the capital of Sicily.
S: Sheboygan is in Wisconsin.
U: Sheboygan is in the USA
D: The Democrats win.
R: The Republicans quarrel.

When we translate statements 4-8 above according this scheme, we have:

9. It is not the case that B.
10. C and L.
11. Either P or M.
12. If S, then U.
13. D if and only if R.

Caveat: There are some statements that count as compound by the above "definition" but which we want to count as atomic (for now), notably:

- Categorical statements:
  - All dogs are mammals.
  - Some plants are trees.
- Statements involving intentional verbs and modals:
  - John believes that Mary is the smartest girl in class.
  - It will always be that Democrats and Republicans quarrel.

The crucial feature that characterizes atomic statements is the lack of anything corresponding to the five logical expressions seen in statements 9-13. These are formalized in the following table:

Operator	**Name**	**Translates**	**Type**
~	tilde	"not"	negation
•	dot	"and"	conjunction
$\vee$	vee	"or"	disjunction
$\rightarrow$	arrow	"if...then"	conditional
$\leftrightarrow$	double-arrow	"if and only if"	biconditional

Negations

The symbol '~', called the *tilde*, is used to translate the English word 'not' and its stylistic variants.

14. Roses are not blue. (R: Roses are blue)

The relevant scheme of abbreviation is given to the right. Thus:

15. ~R

Examples of Stylistic Variants for Negation

- It is not the case that roses are blue.
- It is false that roses are blue.
- It is not true that roses are blue.
- Roses fail to be blue.

All of these are symbolized by (15).

Many negations are negations of compound statements:

16. It is false that Chris is a Buddhist or a Hindu. (B: Chris is a Buddhist. H: Chris is a Hindu)
17. It is not true that if Josh finishes his dissertation this year, he is guaranteed a tenure-track job. (F: Josh finishes his dissertation this year. T: Josh is guaranteed a tenure-track job.)
18. It is not the case that Obama will win and McCain will win. (O: Obama will win. M: McCain will win.)

(16) is the negation of a disjunction; (17) the negation of a conditional; (18) is the negation of a conjunction.

Symbolizing using the given schemes of abbreviation, we have:

19. ~(B ∨ H)
20. ~(F → T)
21. ~(O • M)

Two Important Points

- We use *parentheses* as a form of punctuation.

Consider the result of removing them and the sentences that they in fact symbolize:

22. ~B ∨ H
    - Either Chris is not a Buddhist or he's a Hindu.
23. ~F → T
    - If Josh does not finish his dissertation, then he is guaranteed a tenure track job.
24. ~O • M
    - Obama will not win and McCain will win.

None of the statements so symbolized has the same meaning as the original. Thus:

> Correct "punctuation" by means of parentheses is *absolutely critical* when symbolizing English statements.

- These examples illustrate the difference between the *main* logical operator and *minor* logical operators.

> The *main logical operator* in a compound statement is the one that governs the largest component or components of the statement. A *minor logical operator* in a compound statement governs smaller components.

The tilde is the main logical operator in statements (19)-(21). What are the minor logical operators? What are the main logical operators in (22)-(24)? What are the minor operators?

A More Complex Example of a Negation

25. It is false that, if God is omnipotent and perfectly good, then either horrific suffering is necessary in itself or necessary for some greater good. (P: God is omnipotent. G: God is perfectly good. I: Horrific suffering is necessary in itself. R: Horrific suffering is necessary for some greater good.)

Using this scheme of abbreviation we have:

26. $\sim[(P \bullet G) \rightarrow (I \vee R)]$

Note that we can use *square brackets* instead of parentheses for added clarity. (OK on Web Tutor also.)

Conjunctions

The '•' symbol is used to translate the English word 'and' and its stylistic variants.

27. Hobbes was born in 1588 and Descartes was born in 1596. (H: Hobbes was born in 1588. D: Descartes was born in 1596.)

The relevant scheme of abbreviation is given to the right. Thus:

28. H • D

The statements composing a conjunction are called the *conjuncts* of the conjunction.

Examples of Stylistic Variants for Conjunction

- Hobbes was born in 1588, *but* (*however*) Descartes was born in 1596.
- *While* Hobbes was born in 1588, Descartes was born in 1596.
- *Although* Hobbes was born in 1588, Descartes was born in 1596.
- Hobbes was born in 1588; *yet (nevertheless, nonetheless*) Descartes was born in 1596.
- Hobbes was born in 1588 *even though* Descartes was born in 1596.

A better example for 'even though'.

- Beth is enjoying herself *even though* (or: *despite the fact that*) her boyfriend is crying in the corner.
- The pure logical meaning of "even though" and "despite the fact that" is simply the same as "and".
- However, they signal to the listener/reader that something is amiss; that the two pieces of information are somehow incongruous.

Uses of 'and' Not Translated by '•' : Temporal Succession

29. Stuart climbed Mt Baker and looked inside the sulphur cone.
30. Stuart looked inside the sulphur cone and climbed Mt Baker.

- In a genuine conjunction, the order of the conjuncts doesn't matter.
- It matters here because the order of the conjuncts corresponds to the temporal ordering of the corresponding events.
- Hence, the dot '•' cannot be used to translate this use of 'and'.

Uses of 'and' Not Translated by '•' : Relationships

31. Mike and Kirsten are married.
32. William and Peter are twins.

- Sentences like (31) and (32), in most contexts, indicate *relationships* between the individuals named in the subjects.
- Hence, in those contexts, they cannot be reduced to separate conjunctions ascribing properties to each:
  - "Mike is married and Kirsten is married."
  - The information that Mike and Kirsten are married *to each other* is lost.
  - Hence, again, the dot '•' cannot be used to translate these uses of 'and'.
- Compare "Mike and Kirsten are logicians".
  - Synonymous with "Mike is a logician and Kirsten is a logician."
  - This paraphrase works because "logician" indicates a property true of both Mike and Kirsten individually.

Disjunctions

The '∨' sign is used to translate the English word 'or' and its stylistic variants.

33. Either Carol attends college or she gets a job. (C: Carol attends college. J: Carol gets a job.)

Symbolizing in terms of the scheme of abbreviation:

34. $C \vee J$

The statements composing a disjunction are called the *disjuncts* of the disjunction.

(34) also translates the following stylistic variants:

Examples of Stylistic Variants for Disjunction

- Either Carol attends college *and/or* she gets a job. *(Ick!)*
- Carol attends college *or* she gets a job.
- Either Carol attends college *or* she gets a job (*or both*).
- Carol attends college *unless* she gets a job.

The statements composing a conjunction are called the *disjuncts* of the disjunction.

Remarks on Disjunction

1. Sometimes "or" is used in an *inclusive* sense.

   - "A or B", in this sense, does not rule out the possibility that *both* A and B are true.
   - "The successful applicant must have a BS in Computer Science or five years of programming experience."

2. Sometimes "or" is used in an exclusive sense.

   - "A or B", in this sense, does rule out the possibility that *both* A and B are true.
   - "Either you will apologize to your brother or you will go to your room with no dinner."

3. Following standard practice in logic, we will be taking "or" in its *inclusive* sense.

   - We can express the exclusive sense simply by adding *"but not both*", which we can symbolize with negation (~) and conjunction (•)

35. Either the universe depends for existence on something else or it depends for its existence on nothing, but not both. (S: The universe depends for existence on something else. N: The universe depends for its existence on nothing.)

36. (S ∨ N) • ~(S • N)

> As a general rule, when symbolizing arguments containing disjunctions, assume "or" is used in the inclusive sense unless it renders the argument invalid.

Symbolizing Arguments Involving Disjunction

37. Lassie is either a cat or a dog. Lassie is not a cat. So, Lassie is a dog. (C: Lassie is a cat. D: Lassie is a dog.)

Symbolizing in terms of the scheme of abbreviation:

38. $C \vee D, \sim C \therefore D$

- The vee '$\vee$' is used even though C and D cannot in fact both be true. That is all that is needed for validity.
- Following standard terminology, $\therefore$ is used to mark the conclusion.

Suppose, toward the end of November, someone argues:

39. Either the Aggies will win or UT will win. The Aggies will win. So, UT will not win. (A: The Aggies will win. U: UT will win.)

Symbolizing "or" as inclusive leads to an invalid argument form:

40. $A \vee U,\ A \therefore \sim U$

But in the context, it's a valid argument. So, to represent it fairly, we need to take "or" in the first premise in its exclusive sense:

41. Either the Aggies will win or UT will win, but not it's not the case that both the Aggies and UT will win.

The comma identifies "but" as the main logical operator; thus:

42. $(A \vee U) \bullet \sim(A \bullet U)$

The proper symbolization of the argument, then, is:

43. $(A \vee U) \bullet \sim(A \bullet U),\ A \therefore \sim U$

Two Ways of Symbolizing "Neither..nor"

Statements of the form "Neither A nor B" can be symbolized in two ways:

44. Neither Sue nor Fred is happy. (S: Sue is happy. F: Fred is happy.)

This can be symbolized using either the vee or the dot:

45. ~(S ∨ F)

46. ~S • ~F

Examples of Disjunctions

Each of the following is a disjunction. The main logical operator (by definition) is the vee:

- ~P ∨ Q
- (R • S) ∨ ~T
- (U → W) ∨ ~(X • Y)

Conditionals

The '→' sign is used to translate the English conditionals, typically expressed by "if...then". For example:

47. If Fido is a dog, then he is an animal. (D: Fido is a dog. A: Fido is an animal.)

Symbolizing in terms of the scheme of abbreviation:

48. D → A

The "if" clause of a conditional is known as the *antecedent*. The "then" clause is known as the *consequent*.

(48) also translates the following *stylistic variants*:

- *Given* (*Assuming/Provided*) that Fido is a dog, Fido is an animal.
- Fido is an animal, *given* (*assuming/provided) that* he is a dog.
- Fido is an animal, *if* he is a dog.
- Fido is a dog *only if* he is an animal.
- Fido's being a dog *is a sufficient condition for* Fido's being an animal.
- Fido's being an animal *is a necessary condition for* Fido's being a dog.

A *sufficient* condition for a statement is a condition that guarantees that the statement is true.

A *necessary* condition for a statement is a condition that must hold in order for the statement to be true; that is, a condition that, if lacking, guarantees that the statement is false.

Examples of Conditionals

Each of the following is a conditional. The main logical operator (by definition) is the arrow:

- $\sim X \rightarrow Y$
- $Z \rightarrow (A \vee B)$
- $(C \bullet D) \rightarrow (E \vee \sim F)$

Symbolizing an Argument Involving a Conditional

49. If humans have souls, then immaterial things can evolve from matter. Immaterial things cannot evolve from matter. So, humans do not have souls. (H: Humans have souls. M: Immaterial things can evolve from matter.)

Symbolizing in terms of the scheme of abbreviation:

50. $H \rightarrow M, \sim M \therefore \sim H$

- The form of this argument is *modus tollens*.

Symbolizing 'Unless'

The logical meaning of the word 'unless' is just disjunction:

51. We will lose unless we do our best! (L: We will lose. B: We do our best.)

Thus, the following is fine:

52. $L \vee B$

However, 'unless' has a sort of conditional "feel" to it; notably, (51) seems pretty much identical in meaning to:

51*. We will lose if we don't do our best!

Thus, a somewhat more pleasing symbolization of (51) is:

53. $\sim B \rightarrow L$

As we will shortly be able to prove with truth tables, (52) and (53) are *logically equivalent*; they convey the same information. Hence, both are perfectly acceptable symbolizations of (51).

Biconditionals

The '↔' sign is used to translate English biconditionals, typically expressed by "if and only if". For example:

54. Mary is a teenager *if and only if* she is from 13 to 19 years of age.
    (M: Mary is a teenager. Y: Mary is from 13 to 19 years of age.)

Symbolizing in terms of the scheme of abbreviation:

55. $M \leftrightarrow Y$

The left and right sides of a biconditional are called, well, "the left side of the biconditional" and "the right side of the biconditional"! (They don't have special names.)

(55) also translates the following *stylistic variants*:

- Mary is a teenager *just in case* she is from 13 to 19 years of age.
- Mary's being a teenager is a *necessary and sufficient condition for* Mary's being from 13 to 19 years of age.

Examples of Biconditionals

Each of the following is a biconditional. The main logical operator (by definition) is the double-arrow:

- $\sim X \leftrightarrow Y$
- $\sim Z \leftrightarrow (A \vee B)$
- $(C \bullet D) \leftrightarrow (E \vee \sim F)$

See the text (pp. 288-296) for more examples of symbolization with all the logical operators.

The Formal Grammar of Statement Logic

In this section we provide a more rigorous account of the symbolized sentences, or *well-formed formulas* (*WFFs*) of statement logic.

Lexicon

The *lexicon* of statement logic is its stock of basic symbols. These include

- Statement letters: The capital letters A, B, C, …, Z
- Logical operators: ~, •, ∨, →, ↔
- Parentheses: (,)

Grammar

The *grammar* of statement logic defines the class of syntactically correct expressions of statement logic.

Definition: An *expression* of statement logic is any sequence of symbols from the lexicon of statement of logic.

Examples

- ~ABC~
-))A~•••X↔()C∨→
- (~A↔(B•C))

We will now give the formal grammar of statement logic that defines the WFFs of statement logic. To do so, we will make use of *variables* *p*, *q*, *r*, … that stand for any WFF, just as we might use the variables *m* and *n* to stand for any natural numbers.

Definition (Well-Formed Formula (WFF)):

1. Every statement letter is a *well-formed formula* (*WFF*).
2. If p is a WFF, then so is $\sim p$.
3. If p and q are WFFs, so are $(p \bullet q)$, $(p \vee q)$, $(p \rightarrow q)$, and $(p \leftrightarrow q)$.
4. Nothing counts as a WFF unless it can be demonstrated by to be one by (possibly repeated) applications of 1-3.

The four clauses jointly comprise a *recursive definition* of the notion of a well-formed formula.

Examples and Exposition

Consider the following expressions:

a. PLATYPUS	c. M	e. ~M
b. p	d. (M)	f. (~M)

Which of these are WFFs?

Now consider the following expressions:

g. (M • N)	k. M • N
h. (M ∨ N)	l. M ∨ N
i. (M → N)	m. M → N
j. (M ↔ N)	n. M ↔ N

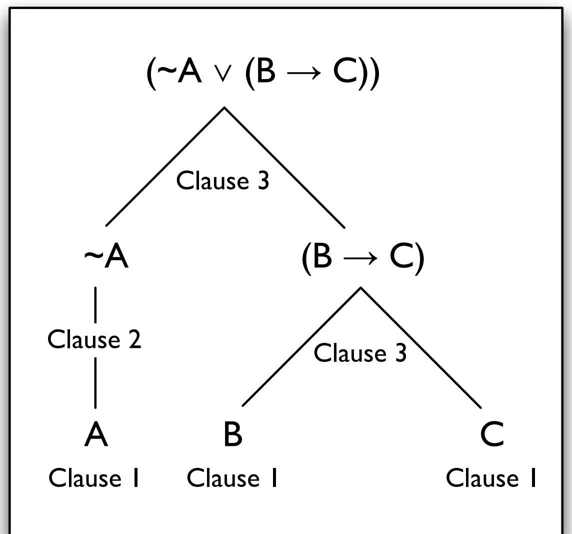
Only those in the left column are genuine WFFs since, by Clause 3 of the definition, WFFs formed from our "binary" operators •, ∨, →, and ↔ have to be surrounded by parentheses.

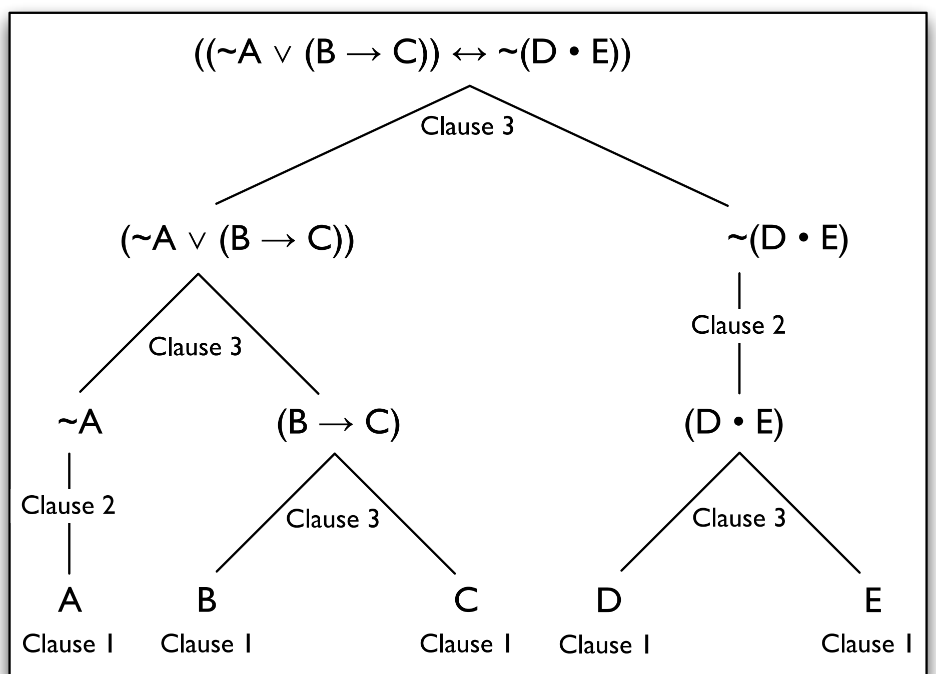
Note how the definition permits the construction of very complex WFFs by iterating of the various clauses, e.g.:

o. (~A ∨ (B → C))

p. ((~A ∨ (B → C)) ↔ (D • E))

We can picture the construction of these WFFs like this:





Acceptable Conventions Regarding Parentheses — Outer Parens

For the sake of convenience, it is *permissible* to drop the outermost parentheses in a WFF because there is no possibility of ambiguity.

- Thus, expressions (k)-(n) above, though not strictly WFFs, are acceptable.
- Note that dropping outermost parentheses is an *acceptable convention.* We have not changed the definition of a WFF. Outermost parentheses in conjunctions, disjunctions, conditionals, and biconditionals are still required for them to count as WFFs.
- The Web Tutor accepts this convention.

It is also permissible (in symbolizing) to drop parentheses from iterated conjunctions and disjunctions in symbolization problems because, again, there is no possibility of ambiguity.

- NB: The Web Tutor WILL NOT accept dropped parentheses in iterated conjunctions/disjunctions.

Examples

	WFF	Acceptable	Unacceptable
q.	((A ∨ B) ∨ C)	A ∨ B ∨ C	—
r.	(A • (B → C))	A • (B → C)	A • B → C
s.	((A • (B • C)) ∨ ~D)	(A • B • C) ∨ ~D	A • B • C ∨ ~D

- The unacceptable expression in line (r) is so because it is ambiguous between A • (B → C) and (A • B) → C, which mean very different things.
- The unacceptable expression in line (s) is so because it is ambiguous between (among others) A • ((B • C) ∨ ~D), (A • B • C) ∨ ~D and (A • B) • (C ∨ ~D).

Square Brackets

It is also acceptable for the sake of clarity to use square brackets instead of parentheses.

	WFF	Acceptable
t.	(A • (B → C))	[A • (B → C)]
u.	((A → (B • C)) ∨ ~D)	(A → [B • C]) ∨ ~D
v.	((A ∨ (B ∨ C)) ↔ ((D • E) ∨ F))	[A ∨ B ∨ C] ↔ [(D • E) ∨ F]

- Note that the second and third examples here involve both of our conventions.
- Again, these are *acceptable conventions*. We have not changed the definition of a WFF. Genuine WFFs can only contain parentheses, not square brackets.
- The Web Tutor WILL acceptable square brackets as alternatives to parentheses.