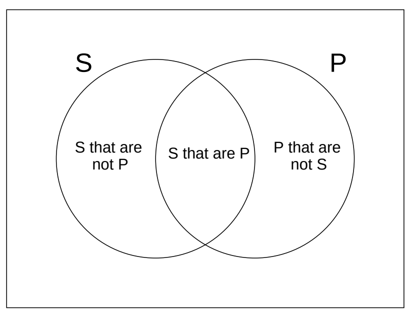
6.2 Venn Diagrams and Categorical Statements

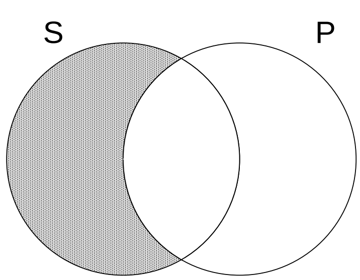
Comment: *Venn diagrams* are a method of determining whether a given categorical syllogism is valid or invalid discovered by John Venn around 1880.

A venn diagram consists of overlapping circles. Each circle stands for a class. Each pair of overlapping circles represents the classes signified by the terms in a categorical statement.

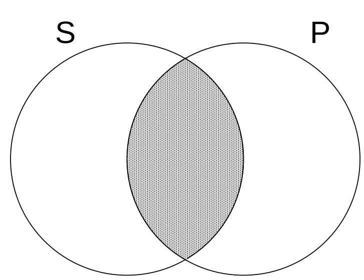
To construct Venn diagrams we first have to be able to diagram each of the four basic forms of categorical statements.



We use *shading* to indicate that a given region (i.e., the class or classes it represents) is empty.

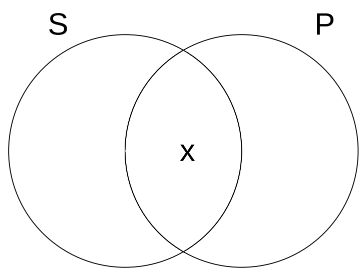


All S are P

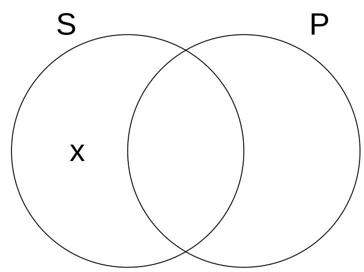


No S are P

We use an X to indicate that a region (i.e., the classes indicated by the region) is nonempty, i.e., contains at least one object.



Some S are P



Some S are not P

Evaluating Arguments: Immediate Inferences

Three steps in evaluating an argument:

1. Diagram the premises.

2. Check whether the diagram contains the content of the conclusion.

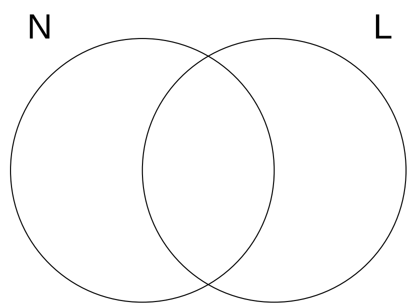
3. If it does, conclude the argument is valid; if not, conclude that it is invalid.

Comment: This method works because, in a deductively valid argument, the content of the conclusion is already contained implicitly in the premises. A Venn diagram of the premise enables us to see this explicitly.

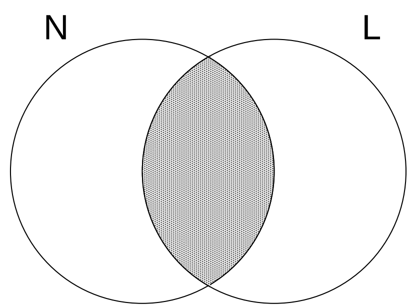
Example 1

1. No Namibians are Libyans.
2. Hence, no Libyans are Namibians.

First draw two overlapping circles, appropriately labeled:



Then diagram as noted for **E** statements:

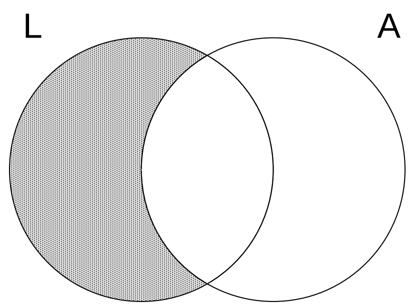


Now, having diagrammed the premise, we look to see if the diagram represents the conclusion. And in this case the answer is yes, as the diagram equally well represents the information that no Libyans are Namibians.

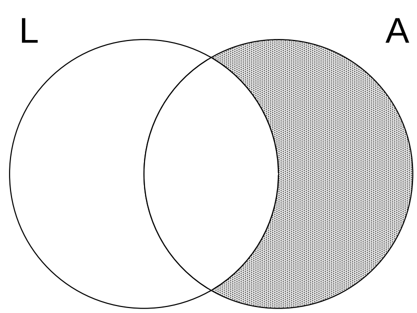
Example 2

1. All Liberians are Africans.
2. Hence, all Africans are Liberians.

The diagrammed premise looks like this:



In this case, however, the content of the conclusion is *not* contained in the premise, as that requires that the area of the **A** circle that does not overlap with the **L** circle to be shaded, i.e.:

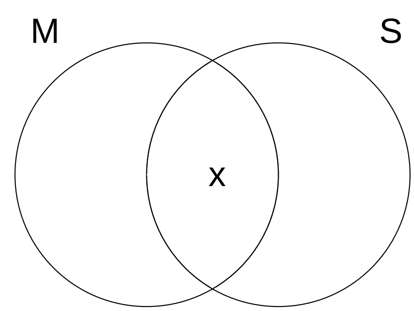


This tells us that the argument is *not* valid. (This should be obvious in this example, as the premise is true and the conclusion false, but the point is that *no* argument of this form is guaranteed to be valid, even if it has true premises and a true conclusion.)

Example 3

1. Some Moroccans are Spanish speakers.
2. Hence, Some Spanish speakers are Moroccans.

Diagramming the premise we have:

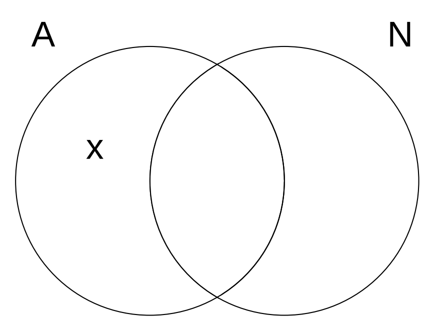


And, as in the case of conversion for **E** statements, we see again that, upon diagramming the premise, the content of the conclusion emerges as well. So the argument is valid.

Example 4

1. Some Africans are not Nigerians.
2. Hence, some Nigerians are not Africans.

Diagramming the premise we have:



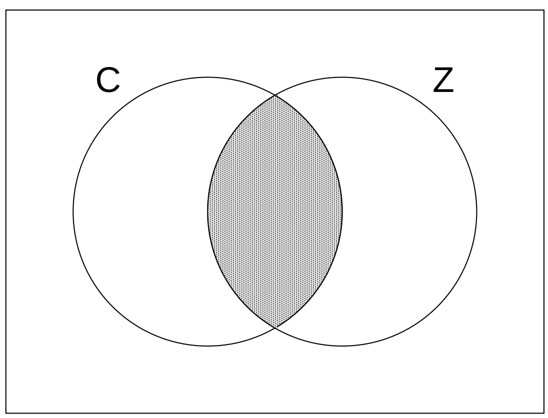
And again, as with **A** statements, we see that the content of the conclusion is not to be found in this diagram, as that would require an X to occur in the are of the **N** circle that is outside the **A** circle — representing thereby (and only thereby) a Nigerian who is not an African.

Example 5

Next we consider an instance of obversion:

1. No Cameroonians are Zimbabweans.
2. Hence, all Cameroonians are non-Zimbabweans.

Since this argument involves a term-complement, it will be useful to frame the diagram of the premise with a box:



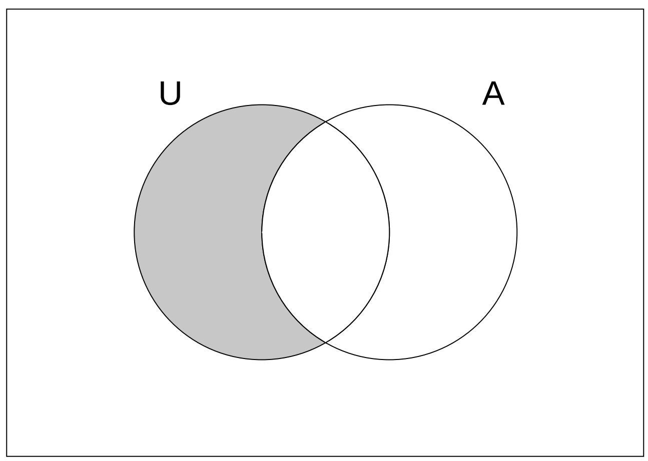
As noted already, all instances of obversion are valid. The diagram helps us to see why. Recall that, in a Venn diagram, the complement of a class is indicated by all unshaded areas *outside* the circle representing the class. Given that there is no overlap between the two classes, as indicated by the shaded area, all the Cameroonians there are must be represented by the unshaded area in the **C** circle. But that area is entirely outside the circle representing the class of Zimbabweans, i.e., that is, it is entirely included in the area representing the class of non-Zimbabweans. Hence, the diagram represents all Camaroonians as non-Zimbabweans, as the conclusion asserts.

Example 6

Next, we consider an instance of contraposition:

1. All Ugandans are Africans.
2. Hence, all non-Africans are non-Ugandans.

To evaluate this argument we again add a box around the diagrammed premise for clarity:



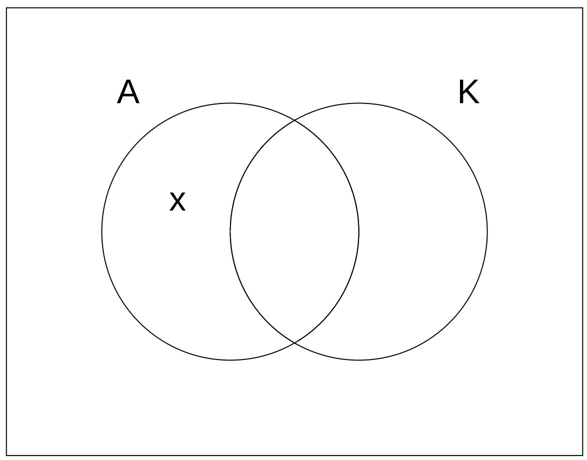
It's a little harder to see at first, but the content of conclusion is present here as well (as it should be, since contraposition is valid for **A** statements). Recalling again that the complement of a class is indicated by all *unshaded* areas *outside* the circle representing the class, we see that, in the diagram above, the class of non-Africans is represented in the diagram by the area of the unshaded area outside the **A** circle. Similarly, the class of non-Ugandans is represented by the entire area of the diagram that is outside the **U** circle. But the latter area includes all of the former. Hence, the diagram of the premise also represents the class of non-Africans as a subclass of the non-Ugandans. I.e., the content of the conclusion is contained in the diagram just in virtue of diagramming the premise, i.e., the argument is valid.

Example 7

Finally, we consider an invalid instance of contraposition:

1. Some Africans are non-Kenyans.
2. Hence, some Kenyans are non-Africans.

Again, we box the diagram for clarity:



Note that we have a choice to represent either the class of Kenyans or the class of non-Kenyans with a circle. In this example we represent the former. The non-Kenyans, then are represented by the area outside of the **K** circle, so to diagram the premise we need an X in the area of the **A** circle that is outside the **K** circle. But for the content of the conclusion to be represented, we would need an X inside the **K** but outside the **A** circle, which we clearly do not have in our diagram of the premise. Hence the content of the conclusion is not contained in the diagram, i.e., the argument is invalid.