1.3 — Counterexamples and Invalidity

In this section, we study a method for determining that some invalid arguments are indeed invalid. This method will therefore reveal cases of *invalid argument forms*:

> An *invalid argument form* is one that has an invalid substitution instance.

Counterexamples

1. If Britney Spears is a philosopher, then Britney Spears is wise.
2. Britney Spears is not a philosopher.

So, 3. Britney Spears is a not wise.

At first site this might seem to be an instance of *modus tollens*:

Modus Tollens

1. If A, then B.
2. Not B.

So, 3. Not A.

But in fact it's an instance of the *invalid* argument form known as the *fallacy of denying the antecedent*:

Fallacy of Denying the Antecedent

1. If A, then B.
2. Not A.

So, 3. Not B.

It is easy to miss the invalidity of the argument above because the conclusion is manifestly true.

But consider the following instance of the same argument form:

1. If Britney Spears is an oil tycoon, then Britney Spears is rich.
2. Britney Spears is not an oil tycoon.

So, 3. Britney Spears is not rich.

Unlike the argument above, this argument is *clearly* invalid because most people (most Americans, at least) will recognize that the premises are true and the conclusion false and hence as a *counterexample* to the fallacious argument form above.

> An *counterexample* to an argument form is a substitution instance whose premises are true and whose conclusion is false.

Not every counterexample is as effective as every other:

1. If there are Beefsteaks in Dan's summer garden, then there are tomatoes in it.
2. There are no Beefsteaks in Dan's summer garden.

So, 3. There are no tomatoes in Dan's summer garden.

This may well be a counterexample; to the fallacy of denying the antecedent, but it isn't obvious unless you know who Dan is and what the condition of his summer garden is.

In general:

The more obvious it is that the premises of a counterexample are true and the conclusion false, the more effective it is!

Thus:

> An *good counterexample* to an argument form is a substitution instance whose premises are *obviously* true and whose conclusion is *obviously* false.

Another Invalid Form

1. If Ryan is a pop-culture buff, then he reads *Entertainment Weekly*.
2. Ryan reads *Entertainment Weekly*.

So, 3. Ryan is a pop-culture buff.

At first site this might seem to be an instance of *modus ponens*:

Modus Ponens

1. If A, then B.
2. A.

So, 3. B.

But in fact it's an instance of the *invalid* argument form known (for obvious reasons) as the *fallacy of affirming the consequent*:

Fallacy of Affirming the Consequent

1. If A, then B.
2. B.

So, 3. A.

Here is a good counterexample:

1. If lemons are red, then lemons have a color.
2. Lemons have a color.

So, 3. Lemons are red.

The Counterexample Method

- A good counterexample to an argument form shows vividly that that form is invalid.
- An argument is *formally invalid*, recall, if it is an instance of an invalid argument form.
- The *counterexample method* (described below) is a method for showing that a given argument is formally invalid by constructing a good counterexample to its argument form.

Note: Recall that any argument whose conclusion cannot be false is valid, so there are valid instances of invalid argument forms. However, such arguments, in reality, are very rare. Hence, *typically*, a formally invalid argument is invalid outright.

Categorical Statements and Arguments (skipping pp. 40-42)

The counterexample method is most useful when applied to arguments whose premises are *categorical statements*.

> An *categorical statement* is a statement that relates two *classes*, or *categories*, of things.

Here is an argument composed of categorical statements:

1. All presidents are human beings.
2. All human beings are mammals.

So, 3. All presidents are mammals.

- The first premise says that the class P of presidents is included in the class H of human beings, i.e., that P is a *subclass* of H.
- The second says that the class H of human beings is included in the class M of mammals, i.e., that H is a subclass of M.
- The conclusion says that the class P of presidents is included in the class of mammals M, i.e., that P is a subclass of M.

Note, however, that this argument is an instance of the following *invalid* form:

Form 5 (Invalid)

1. A.
2. B.

So, 3. C.

However, the argument above is clearly valid and, moreover, it is so in virtue of its form. We just haven't represented its form in the most *logically sensitive* way.

To do so, we need to expand our use of variables to stand for *terms* as well as statements:

> A *term* is a word or phrase that stands for a class.

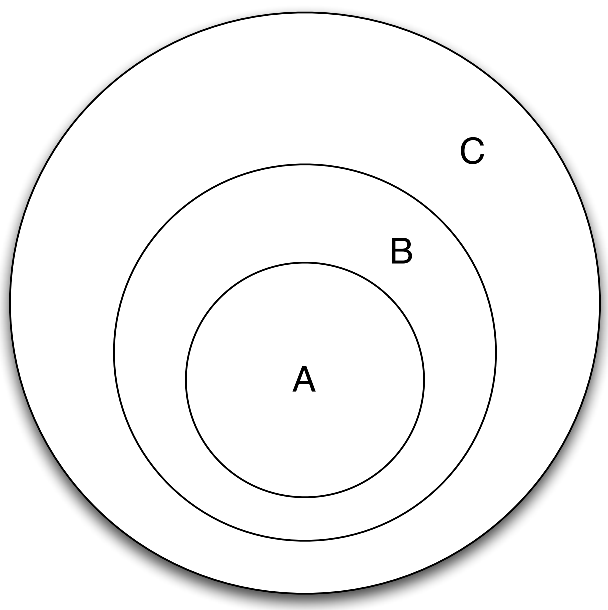
‘presidents’, ‘human beings’, and ‘mammals’ are all terms. Replacing them with ‘A’, ‘B’, and ‘C’, respectively, as we have above yields the following valid argument form:

Form 6 (Valid)

1. All A are B.
2. All B are C.

So, 3. All A are C.

We can illustrate Form 6’s validity with the following diagram:



No matter what classes A, B, and C are, if A is a subclass of B and B a subclass of C, then A *must* be a subclass of C. Any instance of argument form 6 above must therefore be valid.

- The correct form for a valid argument, therefore, is the one that is *duly sensitive to its key logical words and phrases* — notably, ‘all’, ‘some’, ‘no’, ‘if...then’, ‘either...or’, ‘not’, ‘and’, etc.

Further Valid Argument Involving Categorical Statements

1. All emeralds are gems.
2. Some rocks are not gems.

So, 3. Some rocks are not emeralds.

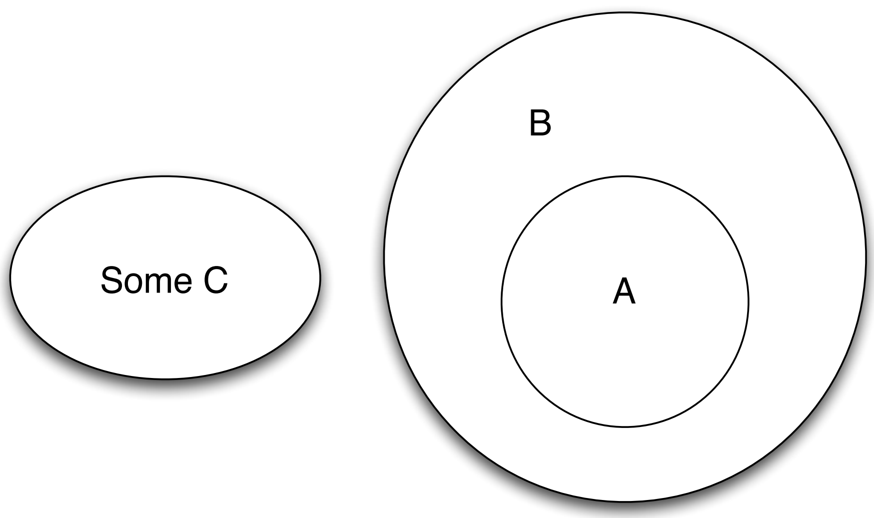
This argument has the following form:

Form 7 (Valid)

1. All A are B.
2. Some C are not B.

So, 3. Some C are not A.

Diagramming, we have:



The diagram clearly illustrates that argument form 7 (and hence its instance above) is valid.

The "Some C" circle indicates simply that some members of the C class fall outside the B class — there might *also* be members of the C class that are also in the B class. But it won't matter either way to the validity of the argument as long as there are at least some members of C that are outside B.

And another:

1. Every sockeye is a member of *Oncorhyncus*.
2. Some natives of the Copper River are sockeye.

So, 3. Some natives of the Copper River are members of *Oncorhyncus*.

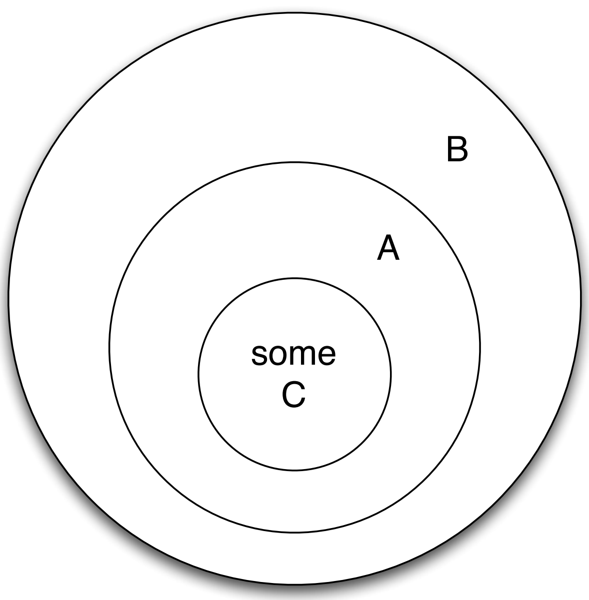
This argument has the following form:

Form 8 (Valid)

1. Every A is a B.
2. Some C are A.

So, 3. Some C are B.

Diagramming, we have:



Again, the diagram clearly illustrates that the argument form in question (and hence its instance above) is valid.

Applying the Counterexample Method

Consider the following argument.

1. All logicians are smart people.
2. Some smart people are not stylish people.

So, 3. Some logicians are not stylish people.

To see if we can find a counterexample, first identify the form:

Form 9

1. All A are B.
2. Some B are not C.

So, 3. Some A are not C.

Next, we try to construct a good counterexample. It is best to start by finding an obviously false conclusion using well understood terms — simple biological species are usually a good choice:

1. All A are B.
2. Some B are not C.

So, 3. Some cheetas are not felines.

Now replace terms uniformly; since we are using 'A' for 'cheetas' and 'C' for 'cats', we have:

1. All cheetas are B.
2. Some B are not felines.

So, 3. Some cheetas are not felines.

Now all we have to do is see whether we can find a term for 'B' that will yield obviously false premises; clearly, we can:

1. All cheetas are animals.
2. Some animals are not felines.

So, 3. Some cheetas are not felines.

One more example:

1. No swimming teachers are aquaphobes.
2. Some bankers are not aquaphobes.

So, 3. Some bankers are not swimming teachers.

Replacing terms with 'A', 'B', and 'C' uniformly, we identify the form:

Form 10

1. No A are B.
2. Some C are not B.

So, 3. Some C are not A.

Let's start with the conclusion and find a counterexample:

1. No __________________ are _____________________.

2. Some _________________ are not __________________.

So, 3. Some _________________ are not __________________.

The Counterexample Method	
Step 1:	Identify the most logically sensitive form of the argument. Use capital letters for variables to stand for statements or terms.
Step 2:	Find English statements or terms that, if substituted for the variables in the conclusion of the argument, yield a well-known falsehood.
Step 3:	Substitute these English statements or terms for the relevant variables uniformly throughout the argument form.
Step 4:	Find English statements or terms that, if substituted uniformly for the remaining variables, produce premises that are well-known truths.
Step 5:	Check your work. If you have succeeded, you have shown that the original argument is *formally* invalid (and, most likely, invalid outright).

A Final Limitation

We might fail to find a counterexample to an argument's form for one of two reasons

- The argument is formally valid
- It is formally invalid, but we simply weren't clever enough to find a counterexample.
- Hence, if we fail to find a counterexample for an argument's form, we cannot infer anything about the argument's formal validity!

We will discover methods that overcome these limitations in the next chapter!